The essence and origins of FRP

or

How you could have invented
Functional Reactive Programming

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LambdaJam 2015

What is FRP?

FRP's two fundamental properties

- Precise, simple denotation. (Elegant & rigorous.)
- *Continuous* time. (Natural & composable.)

Deterministic, continuous "concurrency".

Warning: most modern "FRP" systems have neither property. ☹

FRP's two fundamental properties

- Precise, simple denotation. (Elegant & rigorous.)
- *Continuous* time. (Natural & composable.)

FRP *is not* about:

- graphs,
- updates and propagation,
- streams,
- doing

Why (precise & simple) denotation?

- Separates specification from implementation.
- *Simple* so that we *can* reach conclusions.
- *Precise* so that our conclusions will be *true*.
- Denotations have elegant, functional-friendly style.

An API is a language for communicating about a domain.

It helps to (really) understand what we're talking about.

Why continuous & infinite (vs discrete/finite) time?

Same benefits as for space (vector graphics):

- Transformation flexibility with simple & precise semantics.
- Modularity/reusability/composability:
  - Fewer assumptions, more uses (resolution-independence).
  - More info available for extraction.
- Integration and differentiation: natural, accurate, efficient.
- Quality/accuracy.
- Efficiency (adapative).
- Reconcile differing input sampling rates.

Principle: Approximations/prunings compose badly, so postpone.

See *Why Functional Programming Matters*.

Semantics

Central abstract type: *Behavior a* — a "flow" of values.

Precise & simple semantics:

$$\mu :: Behavior\ a \rightarrow (T \rightarrow a)$$

where $T = \mathbb{R}$ (reals).

Much of API and its specification can follow from this one choice.

Original formulation

API

$$
\begin{aligned}
&\mathit{time} && :: \mathit{Behavior}\ T \\
&\mathit{lift}_0 && :: a \to \mathit{Behavior}\ a \\
&\mathit{lift}_1 && :: (a \to b) \to \mathit{Behavior}\ a \to \mathit{Behavior}\ b \\
&\mathit{lift}_2 && :: (a \to b \to c) \to \mathit{Behavior}\ a \to \mathit{Behavior}\ b \to \mathit{Behavior}\ c \\
&\mathit{timeTrans} && :: \mathit{Behavior}\ a \to \mathit{Behavior}\ T \to \mathit{Behavior}\ a \\
&\mathit{integral} && :: \mathit{VS}\ a \Rightarrow \mathit{Behavior}\ a \to T \to \mathit{Behavior}\ a \\
&\ldots
\end{aligned}
$$

$$
\mathbf{instance}\ \mathit{Num}\ a \Rightarrow \mathit{Num}\ (\mathit{Behavior}\ a)\ \mathbf{where} \ldots
$$

...

Reactivity later.

Semantics

$$
\begin{aligned}
\mu\ time &= \lambda t \rightarrow t \\
\mu\ (lift_0\ a) &= \lambda t \rightarrow a \\
\mu\ (lift_1\ f\ xs) &= \lambda t \rightarrow f\ (\mu\ xs\ t) \\
\mu\ (lift_2\ f\ xs\ ys) &= \lambda t \rightarrow f\ (\mu\ xs\ t)\ (\mu\ ys\ t) \\
\mu\ (timeTrans\ xs\ tt) &= \lambda t \rightarrow \mu\ xs\ (\mu\ tt\ t)
\end{aligned}
$$

$$
\begin{aligned}
&\textbf{instance}\ Num\ a \Rightarrow Num\ (Behavior\ a)\ \textbf{where} \\
&\quad fromInteger = lift_0 \circ fromInteger \\
&\quad (+) \qquad\quad = lift_2\ (+) \\
&\quad \ldots
\end{aligned}
$$

Semantics

$$
\begin{aligned}
\mu\ time &= id \\
\mu\ (lift_0\ a) &= const\ a \\
\mu\ (lift_1\ f\ xs) &= f \circ \mu\ xs \\
\mu\ (lift_2\ f\ xs\ ys) &= liftA_2\ f\ (\mu\ xs)\ (\mu\ ys) \\
\mu\ (timeTrans\ xs\ tt) &= \mu\ xs \circ \mu\ tt
\end{aligned}
$$

$$
\begin{aligned}
&\textbf{instance}\ Num\ a \Rightarrow Num\ (Behavior\ a)\ \textbf{where} \\
&\quad fromInteger = lift_0 \circ fromInteger \\
&\quad (+) \qquad\quad = lift_2\ (+) \\
&\quad \ldots
\end{aligned}
$$

Events

Secondary type:

$$\mu :: \mathit{Event}\ a \to [(T, a)] \quad \text{-- non-decreasing times}$$

$$\begin{aligned}
&\mathit{never} && :: \mathit{Event}\ a \\
&\mathit{once} && :: T \to a \to \mathit{Event}\ a \\
&(.|.) && :: \mathit{Event}\ a \to \mathit{Event}\ a \to \mathit{Event}\ a \\
&(\Longrightarrow) && :: \mathit{Event}\ a \to (a \to b) \to \mathit{Event}\ b \\
&\mathit{predicate} && :: \mathit{Behavior}\ \mathit{Bool} \to \mathit{Event}\ () \\
&\mathit{snapshot} && :: \mathit{Event}\ a \to \mathit{Behavior}\ b \to \mathit{Event}\ (a, b)
\end{aligned}$$

Exercise: define semantics of these operations.

Reactivity

Reactive behaviors are defined piecewise, via events:

$$\mathit{switcher} :: \mathit{Behavior}\ a \to \mathit{Event}\ (\mathit{Behavior}\ a) \to \mathit{Behavior}\ a$$

Semantics:

$$\mu\ (b_0\ \text{`}\mathit{switcher}\text{`}\ e)\ t = \mu\ (\mathit{last}\ (b_0 : \mathit{before}\ t\ (\mu\ e)))\ t$$

$$\mathit{before} :: T \to [(T, a)] \to [a]$$
$$\mathit{before}\ t\ \mathit{os} = [a \mid (t_a, a) \leftarrow \mathit{os}, t_a < t]$$

Important: $t_a < t$, rather than $t_a \leqslant t$.

A more elegant specification for FRP (teaser)

API

Replace operations with standard abstractions where possible:

$$\textbf{instance}\ \mathit{Functor}\ \mathit{Behavior}\ \textbf{where}\ \ldots$$
$$\textbf{instance}\ \mathit{Applicative}\ \mathit{Behavior}\ \textbf{where}\ \ldots$$
$$\textbf{instance}\ \mathit{Monoid}\ a \Rightarrow \mathit{Monoid}\ (\mathit{Behavior}\ a)\ \textbf{where}\ \ldots$$

$$\textbf{instance}\ \mathit{Functor}\ \mathit{Event}\ \textbf{where}\ \ldots$$
$$\textbf{instance}\ \mathit{Monoid}\ a \Rightarrow \mathit{Monoid}\ (\mathit{Event}\ a)\ \textbf{where}\ \ldots$$

Why?

- Less learning, more leverage.
- Specifications and laws for free.

Specifications for free

The *instance's meaning* follows the *meaning's instance*:

$$
\begin{aligned}
\mu\ (\mathit{fmap}\ f\ \ as) &\equiv \mathit{fmap}\ f\ (\mu\ as) \\
\\
\mu\ (\mathit{pure}\ a) &\equiv \mathit{pure}\ a \\
\mu\ (fs \circledast xs) &\equiv \mu\ fs \circledast \mu\ xs \\
\\
\mu\ \varepsilon &\equiv \varepsilon \\
\mu\ (top \diamond bot) &\equiv \mu\ top \diamond \mu\ bot
\end{aligned}
$$

- Corresponds exactly to the original FRP denotation.
- Follows inevitably from a domain-independent principle.
- Laws hold for free.

History

1983–1989 at CMU

- I went for graphics.
- Did program transformation, FP, type theory.
- Class in denotational semantics.

1989 at CMU

- Kavi Arya's visit:
  - *Functional animation*
  - Streams of pictures
  - Elegant, mostly
- John Reynolds's insightful remark:

  "You can think of streams as functions from the natural numbers.
  Have you thought about functions from the *reals* instead?
  Doing so might help with the awkwardness of interpolation."

  Continuous time!

- I finished my dissertation anyway.

1990–93 at Sun: TBAG

- 3D geometry etc as first-class immutable values.
- Animation as immutable functions of continuous time.
- Multi-way constraints on time-functions.
  Off-the-shelf constraint solvers (DeltaBlue & SkyBlue from UW).
- Differentiation, integration and ODEs specified via *derivative*.
  Adaptive Runge-Kutta-5 solver (fast & accurate).
- Reactivity via `assert`/`retract` (high-level but imperative).
- Optimizing compiler via partial evaluation.
- In Common Lisp, C++, Scheme.
- Efficient multi-user distributed execution for free.

1994–1996 at Microsoft Research: RBML/ActiveVRML

- Programming model & fast implementation for new 3D hardware.
- TBAG + denotative/functional reactivity.
- Add event algebra to behavior algebra.
- Reactivity via behavior-valued events.
- Drop multi-way constraints "at first".
- Started in ML as "RBML".
- Rebranded to "ActiveVRML", then "DirectAnimation".

1995–1999 at MSR: RBMH/Fran

- Found Haskell: reborn as "RBMH" (research vehicle).
- Very fast implementation via sprite engine.
- John Hughes suggested using *Arrow*.

1999 at MSR: first try at push-based implementation

- Algebra of imperative event listeners.
- Challenges:
  - Garbage collection & dependency reversal.
  - Determinacy of timing & simultaneity.
  - I doubt anyone has gotten correct.

2009: Push-pull FRP

- Minimal computation, low latency, *provably correct*.
- Push for reactivity and pull for continuous phases.
- "Push" is really blocked pull.
- More elegant API:
  - Standard abstractions.
  - Semantics as homomorphisms.
  - Laws for free.
- Reactive normal form, via equational properties (denotation!).
- Uses *lub* (basis of PL semantics).
- Implementation subtleties & GHC RTS bugs. Didn't quite work.

1996–2014: Paul Hudak / Yale

- Paul Hudak visited MSR in 1996 or so and saw RBMH.
- Encouraged publishing, and suggested collaboration.
- Proposed names “Fran” & “FRP”.
- *Many* FRP-based papers and theses.

July 15, 1952 – April 29, 2015

Questions

“But computers are discrete, ...”